Magnifying Mathematics with Digital Microscopes

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Abstract: *This article describes a series of activities for intermediate-level students to explore ratio reasoning using a digital microscope. In the Common Core State Standards, this subject is addressed for sixth- and seventh-grade students, although this type of reasoning should be introduced at a younger age. Thus, the authors designed this set of activities and then implemented them with a group of fourth-graders. The lesson was well-received and established a foundation for future development of ratio reasoning.*

Keywords. *ratio, proportional reasoning, inquiry*

1 Introduction

In some places, the groundhog is regarded as an annoyance; however, in Punxsutawney, Pennsylvania, this animal is venerated as a hometown mascot. Dubbed "Punxsutawney Phil," the groundhog is honored with 32 fiberglass statues scattered around the town. Each is 6 feet tall and decorated according to various themes, ranging from celebrities to cultural icons. For example, the statue from Punxsutawney shown in Figure 1, sponsored by the US Postal Service, is named Philatelic Phil in reference to philately, the study of postal stamps and other postal history.

Fig. 1: *Philatelic Phil (Kalish, 2010).*

Fiberglass statues like this one are common in many cities around the world. Eugene, Oregon, features ducks; Chicago showcases cows; Hanover, Germany, honors elephants; and Bath, England, has statues of pigs in its commercial district. These statues provide an excellent springboard to learning about ratios and proportions.

On a recent visit to Punxsutawney, my 8 year-old daughter noticed the fiberglass groundhogs, and told me that she thought that the statues were much larger than an actual groundhog. I asked her how much taller she thought they were compared to an actual groundhog. She guessed ten times taller. I thought that was a reasonable guess. According to the National Geographic website (National Geographic, 2015), the average length of a groundhog from its head to its feet is 18-24 inches, thus making the statues between three and four times taller than the average groundhog sitting on its hind legs (See Figure 2). This simple conversation spawned the idea for a class investigation aimed at using proportional reasoning to introduce elementary-level students to the concept of ratios.

Fig. 2: *Actual groundhog (Cooper, 2012).*

2 Real-world proportions

The idea of making things bigger or smaller is a common concept in various fields. Many occupations use scale models in their work. Artists use scaling to recreate a simple piece of fruit on a canvas or to create a statue; architects use scale models in designing a house or building; historians use scale models of the world to study events; and geographers use maps to tie the physical terrain of an area into its surroundings. Even in our daily lives, we deal with objects that are displayed at a size larger or smaller than their actual size. Things as simple as typing a letter on a computer or taking a photograph are often not done in actual size. All of these activities mentioned create a scale model of what viewers see and require viewers to understand ratio reasoning and make conscious or unconscious mental conversions when looking at these objects.

Clearly, many people use ratio reasoning in practical ways. For many math students, the concept of ratios is often difficult to comprehend and for many teachers, teaching this concept presents challenges. Although many science classrooms do these types of conversions, it is valuable to consider the same ideas and topics within the mathematics classroom. This type of reasoning is one of the most important concepts taught in middle school because it provides a foundation for such topics as slope, similarity of shapes, trigonometry, and logarithms (NCTM, 2013). Thus, teachers should give students many opportunities to work with ratios and proportional relationships. Although the Common Core State Standards do not recommend that students understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities until sixth grade, their importance cannot be underestimated and should be introduced in fourth or fifth grade (CCSSI, 2010).

So, how do teachers get students to understand ratios and scale models? The challenge in our schools is to teach students the mathematical concept of ratios within a context that they can enjoy, while at the same time providing them with an opportunity to make observations and conjectures, to take measurements, to notice details, or to use the concept in an everyday situation.

3 Classroom Activities

The following activities provide teachers with some introductory approaches to helping young students understand ratio concepts while intriguing them enough to learn. These were implemented by a student teacher (hereafter referred to as "teacher") with a group of fourth-grade students. The activities can be used individually, as part of a single lesson, or spread over several days. The learning objectives for these activities will allow the students to be able to: measure with precision, determine the ratio of an item under a microscope to its actual size, gain an understanding of how items viewed in the microscope are different from items viewed with the naked eye, and give real world examples of where ratios and proportions might be used.

3.1 Shape Detectives

The teacher began by having students attempt to identify some objects that were shown under different magnifications. Students were asked to look at Figure 3 (left) and guess what they thought they were viewing. Several answers included "a storm," "a beetle," and "a blob." Then, the teacher showed the students Figure 3 (middle), which is the same object, and asked students to identify it. Sample answers included "a person's face," "Stitch" (as in the Lilo and Stitch cartoon character), and "an eye." Finally, the students were asked where they thought this "eye" came from. In response, one student identified it as the image of George Washington's eye imprinted on a dollar bill as shown in Figure 3 (right).

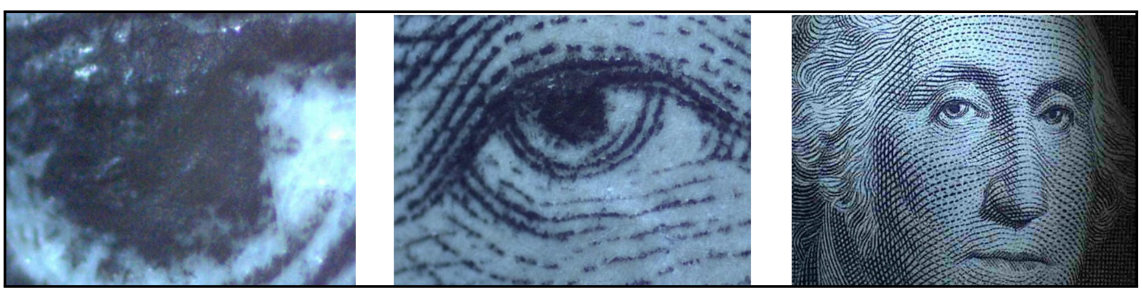

Fig. 3: *Imprinted eye of George Washington on dollar bill at various levels of magnification.*

The teacher continued by asking several follow-up questions: What is the diameter of George Washington's pupil on the dollar bill? Will it always be this size? What do you suppose the ratio is between the pupil on the dollar bill and George Washington's real eye pupil? Do you think his pupil on the dollar bill is the same size as his pupil was in real life? What ratio was used? How could we find out what ratio was used?

4 Under the Microscope

Both George Washington's eye and the fiberglass groundhogs are two of many examples that viewers can see clearly without aid of enlargement and teachers can use for teaching and learning about ratios. An alternative approach lies in enlarging the scale of things that cannot be easily seen with the naked eye or easily measured. Many items are too small for people to see, so they must use a microscope in order to view them. They need to enlarge the objects in order to study them more carefully. Scientists often use microscopes to make these objects "larger." The purpose of a microscope is two-fold. First, a microscope magnifies an object. In other words, it enlarges the object for people to be able to view. Second, a microscope is designed to have a high resolving power. This means that a microscope has the ability to be clearer the closer one gets to the object. This same phenomenon does not hold if one looks at an object with ones own eye. Consequently, using a microscope provides students, especially those at the elementary level, with the perfect opportunity to learn about ratios while making observations, conjecturing, and measuring.

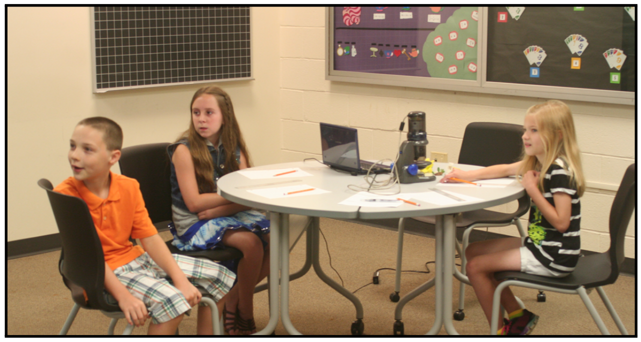

Fig. 4: *Launching microscope activities with a group of early grades students.*

Fig. 5: *The QX-5 digital microscope.*

The mathematical concepts involved in using a digital microscope are simple, and the QX5 has been designed so that the image showing up on the computer is actually 10x (or 60x or 200x) the object. The next activity introduces students to the microscope by using an object they can see and making comparisons of their actual measurement to that on the computer screen with the magnification.

4.1 Measuring Beads

The teacher had students use a metric ruler to measure a bead as shown in Figure 6. As Figure 7 suggests, students were encouraged to be as precise as possible and to check each other's measurements. The teacher collected student responses and asked them to consider how they could determine its width from the microscope's view (under a 10x magnification).

Fig. 6: *Bead at 10x magnification.*

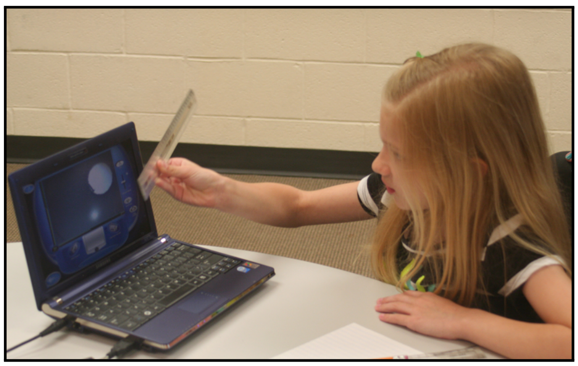

Fig. 7: *Student measuring magnified bead on laptop screen.*

One student replied: "Put two beads across the screen then measure the entire screen and divide by two." Another student replied, "Just measure the bead on the computer screen and divide by ten since it was magnified by ten." Both of these are good replies, and both methods can be used. For this group of students, they used the image on the screen to measure the length of the bead and divided by ten. Then, the students compared their actual measurement to that of the screen. Both came out at 9mm.

The teacher followed up with the question, "What is the point of magnifying the bead and measuring it on the screen if we can just use our ruler to measure it? We got the same answer." Students understood that people usually look at much smaller objects through a microscope and the microscope helps to enlarge them. The teacher then asked students to name some examples of smaller objects that cannot be seen very well with the naked eye. They responded with such examples as "pollen," "water drops," "gnats," and "spider webs." The teacher continued with the lesson by implementing the next activity.

4.2 Under My Thumb

Students were asked to try their own powers of observation and to make a guess as to what the object was in Figure 8. They responded with "bacon," "fish scales," and "fingerprint." Students were then asked to determine a magnification level for the object. In fact, this is a splinter in a person's thumb under 60x magnification (most of the students said 200x).

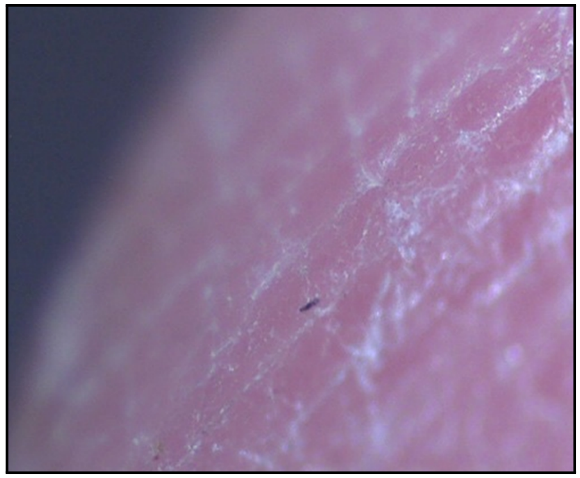

Fig. 8: *Splinter at 60x magnification.*

Finally, the students were asked to work in a group to determine the size of the splinter. They were given both the 60x and 200x magnifications and asked to do both calculations so they could compare answers. Figure 9 illustrates the same object at 200x.

Fig. 9: *Splinter at 200x magnification.*

The students were within $\pm 0.2mm$ when measuring the 60x magnification and within $\pm 0.5mm$ with the 200x magnification. (This splinter measures approximately $1.1mm$). The teacher then said, "Isn't it amazing how something so very small, such as this splinter, can hurt so much?"

Throughout these activities, students were asked to be precise in the measurements, which they did by checking each other's work. They were able to determine the ratio of an item under a microscope to its actual size, as indicated with the splinter. Finally, the students gained an understanding of how items viewed in the microscope are different from items viewed with the naked eye and they considered real world examples of this concept. Students should be further evaluated on their understanding with a worksheet or more practice with the microscope.

4.3 Extending the Activity

If a teacher has more time or students who are interested in learning more about the microscope, he/she could introduce them to the term "field of view" by asking them some leading questions such as: How much are you seeing through the microscope? Why can't you see more? If you change the magnification of the microscope, how does that affect what you see? This is called the *field of view* which is the total area visible through the microscope. Field of view is simply the number of millimeters a spectator will see in one's whole view when looking into the eyepiece lens. It is just as if a person puts a ruler under the microscope and counts the number of lines. This idea is illustrated in Figure 10.

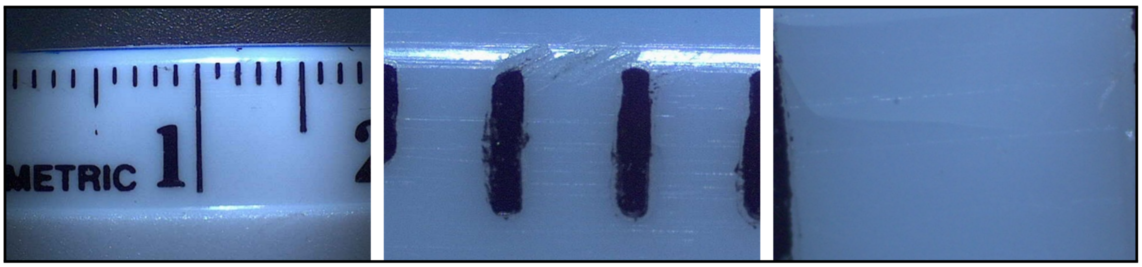

Fig. 10: *A ruler with magnification level of (left) 10x; (middle) 60x; (right) 200x.*

Each image in Figure 10 shows the field of view with a different magnification. In Figure 10 (left), the magnification is 10x and shows 18 millimeters. The image in Figure 10 (middle) is magnified 60 times and shows 3 millimeters and that of Figure 10 (right) is magnified 200 times and shows 1 millimeter. A good question to ask the class is: What happens to the field of view as the magnification gets higher? Students might think that the rectangle stays the same, but the part you see gets smaller. Does it really get smaller? In fact, as the magnification gets larger, the field of view gets smaller. Thus, there is an inverse relationship between the magnification and the field of view. As previously explained, it is possible to measure the width of the field of view. This measurement can then be used to construct reasonable estimates of the size of any specimen or object contained within the field of view.

4.4 Extension of the Bead Activity

Further discussion of the bead activity should lead to explaining that students need to compare the bead to the field of view for 10x. The teacher can ask the students why this was needed and how many beads would fit in this field of view. Students will determine that two beads will fit in our field of view for 10x. Since 10x magnification had a field of view diameter of 18mm, students should be asked what was the width of the bead. In order to determine this, one will use the ratio of the width of the field of view to the number of beads that will fit across the field as shown below:

$$\text{width of bead} = \frac{\text{width of field view}}{\text{\# of beads that will fit across the field}}$$

The bead has a field of view of 18mm. Since two beads fit in the field of view, we have

$$\text{width of bead} = \frac{18mm}{2} = 9mm$$

Note that this is the same measurement as students found in our initial bead activity.

Fig. 11: *Students working on the bead extension task.*

5 Concluding Thoughts

The Common Core State Standards Initiative (CCSSI, 2010) and The National Council of Teachers of Mathematics (NCTM, 2014) emphasize a set of eight Standards for Mathematics Practice (SMP). The activities described here use many of these practices, specifically, modeling with mathematics; attending to precision; using appropriate tools strategically; and looking for and making sense of structure. The CCSS for mathematics (CCSS-M) and the Next Generation Science Standards (NGSS, 2013) as well as individual state mathematics and science standards emphasize the importance of students learning and knowing how to analyze and understand concepts dealing with ratios and proportions. The CCSS-M for 6th grade recommends that "students will be able to analyze simple drawings that indicate the relative size of quantities" and "understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities." Thus, it is essential that students begin to learn about and be exposed to ratio concepts in elementary school.

What is the diameter of George Washington's pupil on the dollar bill? Will it always be this size? What do you suppose the ratio is between the pupil on the dollar bill and George Washington's real eye pupil? These questions and many more can be explored and investigated with a digital microscope. Students have the opportunity to use models and tools to measure and to use ratio reasoning to solve problems. They should also be given the opportunity to bring in their own items for the microscope and explore the artifact's properties on their own. Furthermore, students will become familiar with a simple microscope and know how to use it for other work outside of mathematics.

The activities presented here are offered as introductory activities for teaching ratio reasoning. We hope that the reader will find use in these activities and expand on them for their own classroom. The challenge of getting students to understand the concept of a ratio and be able to use that idea to describe a relationship between two quantities is essential to further mathematical learning and important in our everyday lives.

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